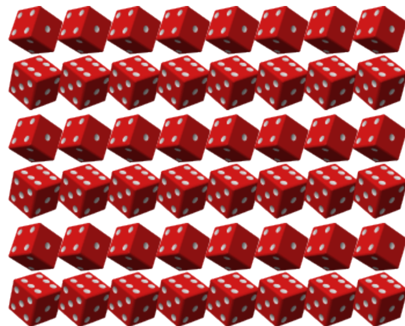


Massively Parallel Random Number Generators

Holger Dammertz, Christoph Schied, Hendrik Lensch

Overview

Pseudo-Random Numbers

- Generated by deterministic algorithms
- In this talk: only pseudo random number generators

Applications for Pseudo-Random Numbers

- Monte Carlo Simulation, Integration
- Test and Content Generation
- These applications are often easily parallelizable

Generating Pseudo-Random Numbers

- Generate i.i.d. uniform random numbers (for example 32bit)
- Transform into $(0, 1)$
- Additional transformation to target distribution (for example, normal distributed)

Structure of an RNG

Formal Definition [L'E06]

$$(S, U, f, g)$$

- S: state space
  - Needed per generating stream (usually per thread)
  - If possible use fast memory
  - Store in global memory after finishing for multiple calls
- $U = (0, 1)$ output space, might need transform from 32 bit integer
- $f : S \to S$ transition function, device function
- $g : S \to U$ output function, device function
- $s_i = f(s_{i-1}), i \geq 1$ and $u_i = g(s_i)$

Structure of an RNG

Example: Linear Congruence Generator LCG [Knu81]

- $s_i \in S$ is an integer (for example 32bit), $g = id, U = S$
- $f : s_i = (as_{i-1} + c) \bmod m$
- Needs well chosen a, c and m

Required properties of an RNG

- Speed
- Repeatability
- Minimal statistical bias, else wrong simulations

Additional properties

- Random access on u_i
- Independent number streams
- Long period

Structure of an RNG

Why is a long period important?

From [SPM05]: for a cycle length of n a single simulation should use at most

$$16\sqrt[3]{n}$$

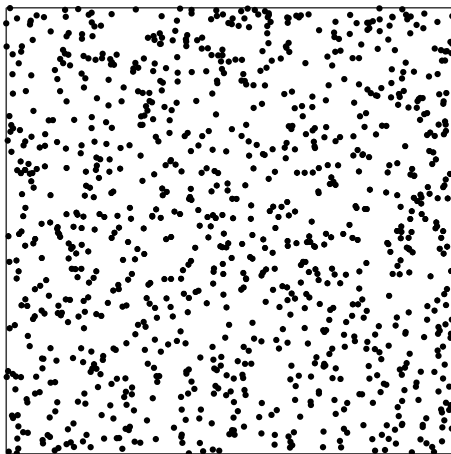
random numbers (to trust the results of statistical simulation).
Assume period of 2^{48} and simple parallel cuda app with 4096 threads:

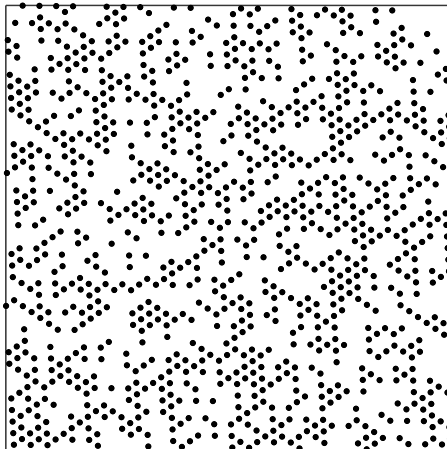
$$\frac{16\sqrt[3]{2^{48}}}{4096} \approx 256$$

random numbers per thread.

Choice of RNG parameters are important

Simple LCG, $2^{10} - 3$ points

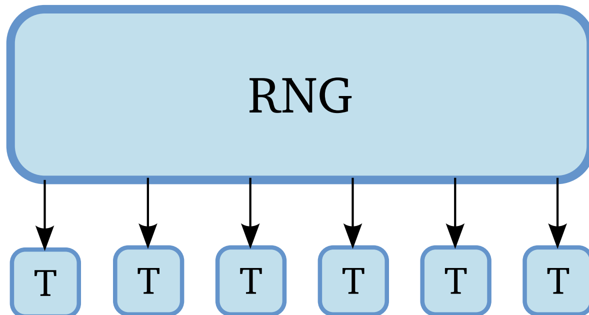




Two different views on parallel random numbers

1) Single Stream for all Threads

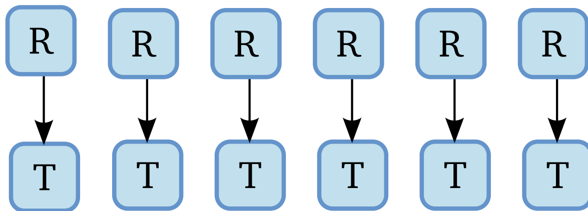
- Each thread computes parts of one problem
- Result should not depend on number of threads and should be repeatable
- Ideally RNG update is also parallel ($\rightarrow$ speed)



Two different views on parallel random numbers

2) Each Thread uses it's own RNG

- Each thread computes individual solutions
- Need to guarantee independence of streams



- If you could have a true RNG both methods would behave exactly the same (but repeatability would be lost).
- Using Pseudo RNGs this needs to be explicitly designed in the program

Parallelizing RNGs

Pseudo-Random Seeding

- Easy to implement
- Generally very bad parallelization method
  - Need to seed valid states
  - No guarantee of independence
- Can work for generators with a long period
- Better alternative: well chosen seeds (per thread)
  - For example: Mersenne Twister dcmt library
  - New GPU Mersenne Twister (MTGP) provides seed tables

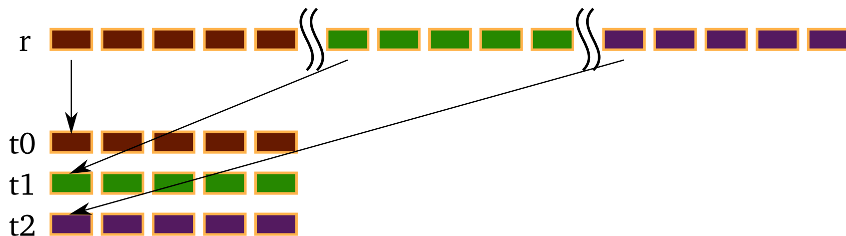
Parametrization

- n different RNG configurations for n threads
- Needs to be especially developed and tested
- Often restricted to specific n

Parallelizing RNGs

Block Splitting

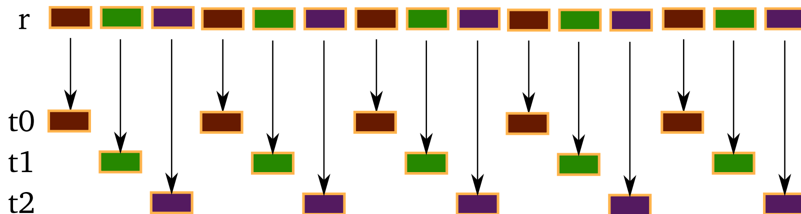
- Can guarantee non overlapping sequences of length m
- m needs to be known in advance
- Starting states need to be known or computed



Parallelizing RNGs

Leap-Frogging

- Needs RNG to be able to skip n numbers (or else quite inefficient)



Application Design Considerations

On the fly Computation

Compute RN when needed in kernel

- State needs to be stored per stream
- RNG uses additional resources (registers and memory)
- Needs properly parallelized implementation

Pre-computation

Store RN in main memory

- Only memory access per RN
- Easily parallelizable (same access as leap frog)
- Memory requirements can be huge
- Bandwidth need to be considered

Upload vs. Computation Example

Intel(R) Xeon(R) E5420 @ 2.50GHz, GTX 480, 256MB random numbers, Mersenne Twister: MTGP/SFMT, 2^{14} threads, blocksize 256, simple Asian Option application

CPU Precomputation

- SFMT: 180ms
- Upload: 44ms
- Consume: 783.5ms
- Total: 1008ms

GPU Precomputation

- MTGP: 9.18ms
- Consume: 783.5ms
- Total: 793ms

GPU On The Fly

- Single Kernel: 1058.1ms

Overview of some RNGs

LCGs

$$s_i = (a s_{i-1} + c) \bmod m$$

- Combining multiple LCGs can give longer period
- Independent streams: Wichmann-Hill (273 threads)

Multiple Recursive Generator

$$s_i = \left(\sum_{\xi=1}^{k} a_\xi \, s_{i-\xi} \right) \bmod m$$

- Larger period (for k=1 equal to LCG)
- Blocking: MRG32k3a [LSCK02]

Overview of some RNGs

RNGs based on Cryptographic functions

- Creates white noise from input
- MD5 [TW08]: hash function
- Tiny Encryption Algorithm (TEA) [ZOC10]
- Different configuration per thread (parametrization)
    - Transform counter and thread id
- Random Access in the sequence

Mersenne Twister

- New GPU version [Sai10]
- Long period: 32bit version provides $2^{11213} - 1, 2^{23209} - 1, 2^{44497} - 1$
- Good seeding strategies (see MTGPDC)

Testing RNGs

Why use tests

- Implementation of RNGs is very sensitive
- Before using any RNG implementation it should be tested
- Failed tests: very likely bad sequence
- Passed tests: guarantees nothing

Test Suits

- Use statistical tests to find flaws
- DIEHARD + NIST (both integrated in DIEHARDER [Bro09])
- TestU01 [LS07]

Collection of Parallel Random Number Generators

Selecting suitable RNGs

- Domain specific problem
- Depends on current compiler and hardware

Our Collection: `http://mprng.sf.net`

- CUDA Linux implementation of several different RNGs
- Copy-paste ready, MIT licensed code
- Easy to extend

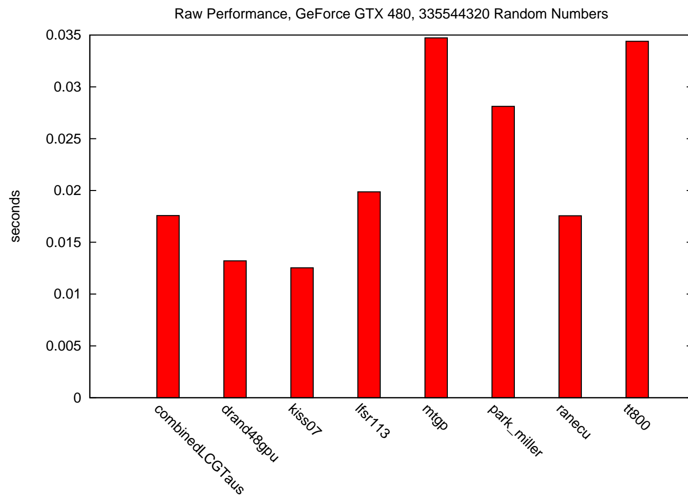
Integrated

- Benchmarks compile and run on your machine
- Configure threads and blocks according to your target application
- DIEHARDER and TestU01 test suites with report generation

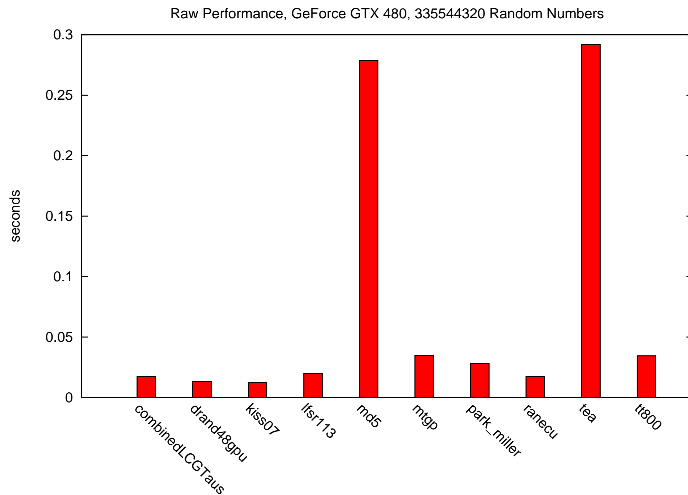
Comparisons: TestU01 SmallCrush Battery

RNG Name	BirthdaySpacings	Collision	Gap	SimpPoker	CouponCollector	MaxOft	MaxOft AD	WeightDistrib	MatrixRank	HammingIndep	RandomWalk1 H	RandomWalk1 M	RandomWalk1 J	RandomWalk1 R	RandomWalk1 C
combinedLCGTaus	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
drand48gpu	P	P	P	P	P	F	F	P	P	P	F	F	F	F	P
kiss07	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
lfsr113	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
md5	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
mtgp	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
park_miller	F	F	P	P	P	F	F	P	P	P	F	F	F	F	F
ranecu	P	F	P	P	P	F	F	P	P	P	F	F	F	F	F
tea	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
tt800	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P

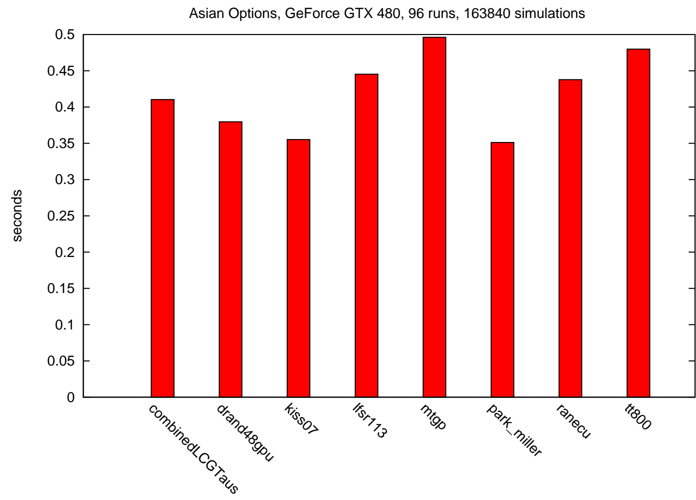
Raw Performance Test



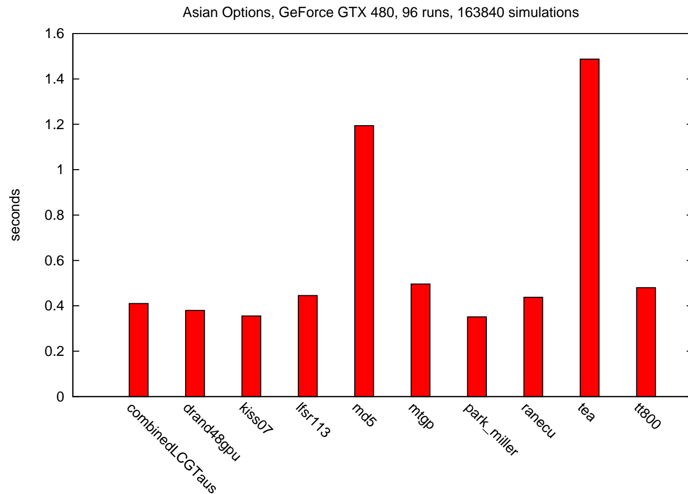
Raw Performance Test



Performance Test: Asian Option Example from [HT07]



Performance Test: Asian Option Example from [HT07]

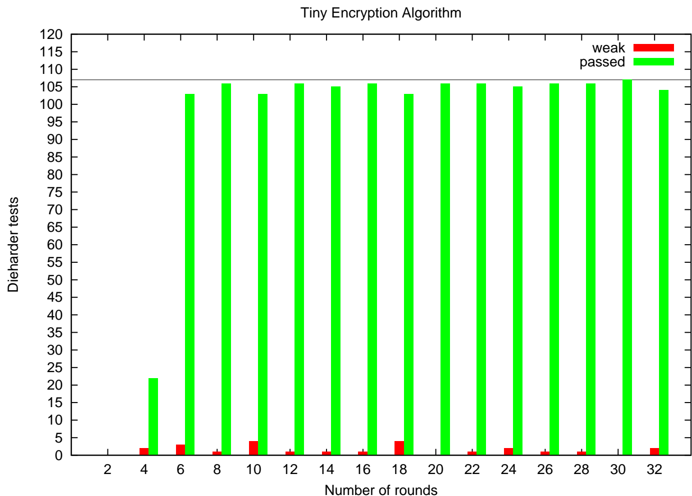


Quality vs. Speed

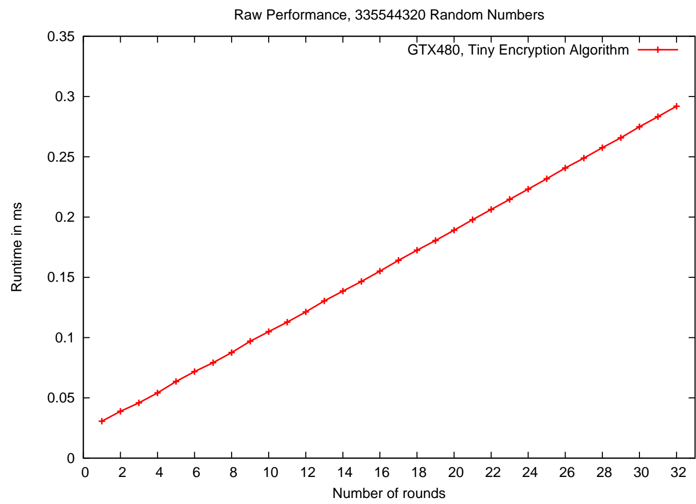
Reduced rounds of MD5/TEA RNG

- For some applications speed more important than quality
- MD5 and TEA allow to reduce number of iterations
  - Quality of random numbers may degrade
  - Avalanche effect
- MD5: passes most of the DIEHARDER tests after 16 rounds

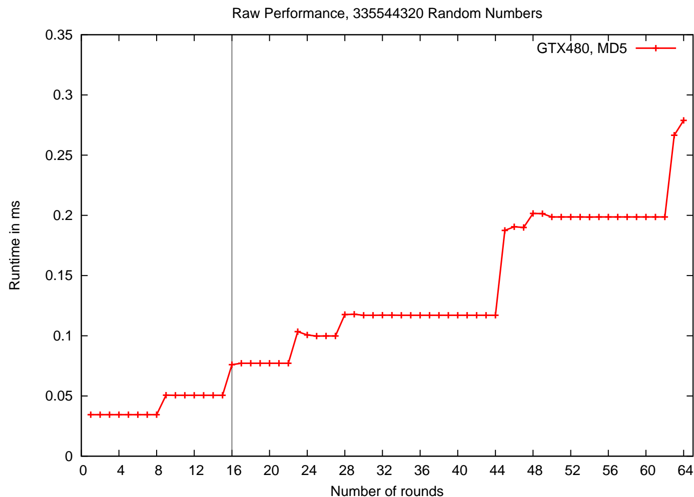
Quality vs. Speed, TEA, DIEHARDER tests



Quality vs. Speed, TEA performance



Quality vs. Speed, MD5 performance



Conclusion

- Many good (parallel) RNGs exist with several different properties
- Choice of fitting RNG application dependent

Some Picks

- KISS07
  - + Simple code and state management
  - – Random seeding: may be ok for non-critical applications
  - – Relatively short Period of 2^{121}
- MTGP
  - + Very good quality and sophisticated seeding
  - + Long period
  - – Relatively complex code and higher resource usage
  - – Fixed block/thread layout
- MD5, TEA
  - + Random access
  - + No Seeding
  - – Slow

Conclusion

Precomputing on GPU

- May be an alternative to in kernel computation

RNG Collection

- Always evaluate your RNG choice and implementation
- Our framework provides an easy platform for testing
  `http://mprng.sf.net`

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Questions?

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